MateMATURA #5 Rozwiązania

Warto wiedzieć (tablice maturalne str. 5)

- Niech $a > 0$ i $a \neq 1$. Logarytmem $\log_a b$ liczby $b > 0$ przy podstawie a nazywamy wykładnik c potęgi, do której należy podnieść a, aby otrzymać b:

$$\log_a b = c \quad \text{wtedy i tylko wtedy, gdy} \quad a^c = b$$

zatem $\log_a a^c = c$

tablice str. 6 → Zapisy $\log x$ oraz $\lg x$ oznaczają $\log_{10} x$.

Zad. 1. Oblicz:

a) $log_5 1 = \log_5 5^0 = 0$ $\qquad 1 = 5^0$

b) $log_2 2 = \log_2 2^1 = 1$ $\qquad 2 = 2^1$

c) $log_7 7 = \log_7 7^1 = 1$ $\qquad 7 = 7^1$

d) $log_5 25 = \log_5 5^2 = 2$ $\qquad 25 = 5^2$

e) $log_2 8 = \log_2 2^3 = 3$ $\qquad 8 = 2^3$

f) $log_5 125 = \log_5 5^3 = 3$ $\qquad 125 = 5^3$

g) $log_6 36 = \log_6 6^2 = 2$ $\qquad 36 = 6^2$

h) $log_4 64 = \log_4 4^3 = 3$ $\qquad 64 = 4^3$

i) $log\,100 = \log_{10} 10^2 = 2$ $\qquad 100 = 10^2$

str. 5 → – dla $a \neq 0$: $a^{-n} = \frac{1}{a^n}$ oraz $a^0 = 1$

Zad. 2. Oblicz:

a) $log_5 \frac{1}{125} = \log_5 5^{-3} = -3$ $\qquad \frac{1}{125} = \frac{1}{5^3} = 5^{-3}$

b) $log_2 \frac{1}{16} = \log_2 2^{-4} = -4$ $\qquad \frac{1}{16} = \frac{1}{2^4} = 2^{-4}$

c) $log_3 \frac{1}{27} = \log_3 3^{-3} = -3$ $\qquad \frac{1}{27} = \frac{1}{3^3} = 3^{-3}$

d) $log_4 \frac{1}{16} = \log_4 4^{-2} = -2$ $\qquad \frac{1}{16} = \frac{1}{4^2} = 4^{-2}$

e) $log_7 \frac{1}{49} = \log_7 7^{-2} = -2$ $\qquad \frac{1}{49} = \frac{1}{7^2} = 7^{-2}$

f) $log\,0,001 = \log_{10} 10^{-3} = -3$ $\qquad 0,001 = \frac{1}{1000} = \frac{1}{10^3} = 10^{-3}$

str. 5 → – dla $a \geq 0$: $a^{\frac{m}{n}} = \sqrt[n]{a^m}$

Zad. 3. Oblicz:

a) $log_2\sqrt{2} = log_2 2^{\frac{1}{2}} = \frac{1}{2}$ $\quad \sqrt{2} = \sqrt[2]{2^1} = 2^{\frac{1}{2}}$

b) $log_2\sqrt[5]{2} = log_2 2^{\frac{1}{5}} = \frac{1}{5}$ $\quad \sqrt[5]{2} = \sqrt[5]{2^1} = 2^{\frac{1}{5}}$

c) $log_2\sqrt[5]{8} = log_2 2^{\frac{3}{5}} = \frac{3}{5}$ $\quad \sqrt[5]{8} = \sqrt[5]{2^3} = 2^{\frac{3}{5}}$

d) $log_7\sqrt[6]{49} = log_7 7^{\frac{2}{6}} = \frac{2}{6} = \frac{1}{3}$ $\quad \sqrt[6]{49} = \sqrt[6]{7^2} = 7^{\frac{2}{6}}$

e) $log\ \sqrt[7]{1000} = log_{10} 10^{\frac{3}{7}} = \frac{3}{7}$ $\quad \sqrt[7]{1000} = \sqrt[7]{10^3} = 10^{\frac{3}{7}}$

f) $log_3\sqrt[4]{27} = log_3 3^{\frac{3}{4}} = \frac{3}{4}$ $\quad \sqrt[4]{27} = \sqrt[4]{3^3} = 3^{\frac{3}{4}}$

Zad. 4. Oblicz:

a) $log\ \sqrt[5]{0,001} = log_{10} 10^{-\frac{3}{5}} = -\frac{3}{5}$ $\quad \sqrt[5]{0{,}001} = \sqrt[5]{\frac{1}{1000}} = \sqrt[5]{\frac{1}{10^3}} = \sqrt[5]{10^{-3}} = 10^{-\frac{3}{5}}$

b) $log_2\frac{1}{\sqrt[5]{4}} = log_2 2^{-\frac{2}{5}} = -\frac{2}{5}$ $\quad \frac{1}{\sqrt[5]{4}} = \frac{1}{\sqrt[5]{2^2}} = \frac{1}{2^{\frac{2}{5}}} = 2^{-\frac{2}{5}}$

c) $log_2\frac{1}{\sqrt{2}} = log_2 2^{-\frac{1}{2}} = -\frac{1}{2}$ $\quad \frac{1}{\sqrt{2}} = \frac{1}{\sqrt[2]{2^1}} = \frac{1}{2^{\frac{1}{2}}} = 2^{-\frac{1}{2}}$

Zad. 5. Oblicz:

a) $log_{\frac{2}{3}}\frac{4}{9} = log_{\frac{2}{3}} \left(\frac{2}{3}\right)^2 = 2$ $\quad \frac{4}{9} = \left(\frac{2}{3}\right)^2$

b) $log_{\frac{2}{3}}\frac{3}{2} = log_{\frac{2}{3}} \left(\frac{2}{3}\right)^{-1} = -1$ $\quad \frac{3}{2} = \left(\frac{3}{2}\right)^1 = \left(\frac{2}{3}\right)^{-1}$

c) $log_{\frac{1}{3}} 9 = log_{\frac{1}{3}} \left(\frac{1}{3}\right)^{-2} = -2$ $\quad 9 = 3^2 = \left(\frac{1}{3}\right)^{-2}$

d) $log_{\sqrt{2}} 4\sqrt{2} = log_{\sqrt{2}} \sqrt{2}^5 = 5$ $\quad 4\sqrt{2} = 2\cdot 2\cdot\sqrt{2} = \sqrt{2}^2\cdot\sqrt{2}^2\cdot\sqrt{2}^1 = \sqrt{2}^5$

e) $log_{\sqrt{6}} 216 = log_{\sqrt{6}} \sqrt{6}^6 = 6$ $\quad 216 = 6^3 = (\sqrt{6}^2)^3 = \sqrt{6}^6$

f) $log_{\sqrt{5}}\frac{1}{25} = log_{\sqrt{5}} \sqrt{5}^{-4} = -4$ $\quad \frac{1}{25} = \frac{1}{5^2} = 5^{-2} = (\sqrt{5}^2)^{-2} = \sqrt{5}^{-4}$

Tablice maturalne (str. 6)

Wzór na zamianę podstawy logarytmu:

jeżeli $a > 0$, $a \neq 1$, $b > 0$, $b \neq 1$ oraz $c > 0$, to:

$$\boxed{\log_b c = \frac{\log_a c}{\log_a b}}$$

Zad. 6. Oblicz

a) $log_4 32 = \frac{log_2 32}{log_2 4} = \frac{log_2 2^5}{log_2 2^2} = \frac{5}{2}$ $\quad 32 = 2^5$, $4 = 2^2$

II sposób (z wzoru: $\log_a b = c$ wtedy i tylko wtedy, gdy $a^c = b$

Zad. 6. Oblicz

a) $log_4 32 = c$ wyłącznie, gdy $4^c = 32$

$(2^2)^c = 2^5$

$2^{2c} = 2^5$

$2c = 5 \; /\!/ 2$

$c = \frac{5}{2}$.

b) $log_{25} 125 = \frac{\log_5 125}{\log_5 25} = \frac{\log_5 5^3}{\log_5 5^2} = \frac{3}{2}$

II sposób: $\log_{25} 125 = c \iff 25^c = 125$

$5^{2c} = 5^3$

$2c = 3 \rightarrow c = \frac{3}{2}$

c) $log_{49} 7 = \frac{\log_7 7}{\log_7 49} = \frac{\log_7 7^1}{\log_7 7^2} = \frac{1}{2}$

II sposób: $\log_{49} 7 = c \iff 49^c = 7 \iff 7^{2c} = 7^1$

$2c = 1 \iff c = \frac{1}{2}$.

d) $log_{\sqrt{2}} \sqrt[7]{8} = \frac{\log_2 \sqrt[7]{8}}{\log_2 \sqrt{2}} = \frac{\log_2 2^{\frac{3}{7}}}{\log_2 2^{\frac{1}{2}}} = \frac{\frac{3}{7}}{\frac{1}{2}} = \frac{3}{7} : \frac{1}{2} = \frac{3}{7} \cdot \frac{2}{1} = \frac{6}{7}$.

II sposób: d) $log_{\sqrt{2}} \sqrt[7]{8} = c \iff \sqrt{2}^c = \sqrt[7]{8}$

$\sqrt[2]{2^1}^c = \sqrt[7]{2^3}$

$2^{\frac{1}{2}c} = 2^{\frac{3}{7}}$

$\frac{1}{2}c = \frac{3}{7} \; / \cdot 2 \rightarrow c = \frac{6}{7}$.

e) $log_9 27 = \frac{\log_3 27}{\log_3 9} = \frac{\log_3 3^3}{\log_3 3^2} = \frac{3}{2}$

II sposób: e) $log_9 27 = c \iff 9^c = 27 \iff 3^{2c} = 3^3$

$2c = 3$

$c = \frac{3}{2}$

f) $log_9 \sqrt{3} = \frac{\log_3 \sqrt{3}}{\log_3 9} = \frac{\log_3 3^{\frac{1}{2}}}{\log_3 3^2} = \frac{\frac{1}{2}}{2} = \frac{1}{2} : 2 = \frac{1}{2} \cdot \frac{1}{2} = \frac{1}{4}$.

II sposób: f) $log_9 \sqrt{3} = c \iff 9^c = \sqrt{3}$

$3^{2c} = \sqrt[2]{3^1}$

$3^{2c} = 3^{\frac{1}{2}}$

$2c = \frac{1}{2} \; /\!/ 2$

$c = \frac{1}{4}$.

tablice matematyczne str. 5

Dla dowolnych liczb rzeczywistych $x > 0$, $y > 0$ oraz r prawdziwe są równości:

1 $\log_a(x \cdot y) = \log_a x + \log_a y$ 2 $\log_a x^r = r \cdot \log_a x$

3 $\log_a\left(\frac{x}{y}\right) = \log_a x - \log_a y$

Zad.7. Oblicz:

1 a) $log_6 9 + log_6 4 = \log_6(9\cdot4) = \log_6 36 = \log_6 6^2 = 2$

1 b) $log\ 20 + log\ 5 = \log(20\cdot5) = \log 100 = \log_{10}10^2 = 2$

1 c) $log_3 \frac{9}{4} + log_3 \frac{4}{27} = \log_3\left(\frac{\cancel{9}^{1}}{\cancel{4}_{1}} \cdot \frac{\cancel{4}^{1}}{\cancel{27}_{3}}\right) = \log_3 \frac{1}{3} = \log_3 3^{-1} = -1$

3 d) $log_2 7 - log_2 56 = \log_2 \frac{7}{56} = \log_2 \frac{1}{8} = \log_2 2^{-3} = -3$

3 e) $log_3 15 - log_3 45 = \log_3 \frac{15}{45} = \log_3 \frac{1}{3} = \log_3 3^{-1} = -1$

3 f) $log\ 7 - log\ 700 = \log \frac{7}{700} = \log \frac{1}{100} = \log_{10} 10^{-2} = -2$

Zad.8. Wyznacz x:

$\log_a b = c \Leftrightarrow a^c = b$

a) $5 = log_2 x$

$x = 2^5 = 32$

b) $3 = log\ x$

$3 = \log_{10} x$

$x = 10^3 = 1000$

c) $-2 = log_5 x$

$x = 5^{-2} = \frac{1}{25}$

d) $\frac{2}{3} = log_8 x$

$x = 8^{\frac{2}{3}} = \sqrt[3]{8}^2 = 2^2 = 4$

e) $\frac{1}{2} = log_9 x$

$x = 9^{\frac{1}{2}} = \sqrt[2]{9}^1 = \sqrt{9} = 3$

f) $2 = log_x 16$

$x^2 = 16$

$x = \sqrt{16} = 4$

g) $\frac{1}{2} = log_x 5$

$x^{\frac{1}{2}} = 5$

$\sqrt{x} = 5$

$x = 5^2 = 25$

h) $-3 = log_x 8$

$x^{-3} = 8$

$x^3 = \frac{1}{8}$

$x = \sqrt[3]{\frac{1}{8}} = \frac{1}{2}$.

i) $-\frac{3}{2} = log_x 8$

$x^{-\frac{3}{2}} = 8$

$x^{\frac{3}{2}} = \frac{1}{8}$

$\sqrt[2]{x^3} = \frac{1}{8}$

$x^3 = \left(\frac{1}{8}\right)^2 = \frac{1}{64}$

$x = \sqrt[3]{\frac{1}{64}} = \frac{1}{4}$.

Zad.8. Zapisz jako jeden logarytm:

1 a) $log_2 3 + log_2 7 = log_2(3 \cdot 7) = log_2 21$

3 b) $log_3 7 - log_3 4 = log_3 \frac{7}{4}$

c) $log_2 3 + 4log_2 2 = 2 = log_2 3 + log_2 2^4 = log_2 3 + log_2 16 =$

$1 = log_2(3 \cdot 16) = log_2 48$.

d) $2log_3 2 - 3log_3 3 = 2 = log_3 2^2 - log_3 3^3 = log_3 4 - log_3 27 = 3 = log_3 \frac{4}{27}$

e) $3 + log\ 7 = log_{10} 10^3 + log 7 = log 1000 + log 7 = log(1000 \cdot 7) = log 7000$

f) $2 - 3\ log_3 4 = log_3 3^2 - log_3 4^3 = log_3 9 - log_3 64 = log_3 \frac{9}{64}$

g) $2\ log_3 5 - 1 = log_3 5^2 - log_3 3^1 = log_3 25 - log_3 3 = log_3 \frac{25}{3}$

h) $3\ log_2 4 - 2 = log_2 4^3 - log_2 2^2 = log_2 64 - log_2 4 = log_2 \frac{64}{4} = log_2 16$.

Dodatkowo

Zad.8. Wiedząc, że $log_3 7 = 1,77$ oraz $log_3 2 = 0,63$ oblicz:

a) $log_3 14 = log_3(2 \cdot 7) = log_3 2 + log_3 7 = 0,63 + 1,77 = 2,4$

b) $log_3 3,5 = log_3 \frac{7}{2} = log_3 7 - log_3 2 = 1,77 - 0,63 = 1,14$

c) $log_3 49 = \log_3 7^2 = 2 \cdot \log_3 7 = 2 \cdot 1{,}77 = 3{,}54$

d) $log_3 21 = \log_3 (3 \cdot 7) = \log_3 3 + \log_3 7 = 1 + 1{,}77 = 2{,}77$

e) $log_3 \frac{1}{8} = \log_3 2^{-3} = -3 \cdot \log_3 2 = -3 \cdot 0{,}63 = -1{,}89$

f) $log_3 \sqrt[7]{8} = \log_3 2^{\frac{3}{7}} = \frac{3}{7} \cdot \log_3 2 = \frac{3}{7} \cdot 0{,}63 = \frac{3}{\cancel{7}_1} \cdot \frac{\cancel{63}^{9}}{100} = \frac{27}{100} = 0{,}27$

g) $log_3 28 = \log_3 (4 \cdot 7) = \log_3 4 + \log_3 7 = \log_3 2^2 + 1{,}77 =$

$2 \cdot \log_3 2 + 1{,}77 = 2 \cdot 0{,}63 + 1{,}77 = 1{,}26 + 1{,}77 = 3{,}03$

h) $log_3 98 = \log_3 (2 \cdot 49) = \log_3 2 + \log_3 49 = 0{,}63 + \log_3 7^2 = 0{,}63 + 2 \cdot \log_3 7 =$

$= 0{,}63 + 2 \cdot 1{,}77 = 0{,}63 + 3{,}54 = 4{,}17$

i) $log_3 42 = \log_3 (2 \cdot 3 \cdot 7) = \log_3 2 + \log_3 3 + \log_3 7 = 0{,}63 + 1 + 1{,}77 = 3{,}4.$