

Umiejętność: II.1. Uczeń stosuje wzory skróconego mnożenia na: $(a + b)^2$, $(a - b)^2$, $a^2 - b^2$;

Warto wiedzieć:
Z tablic maturalnych (str. 7)

6. WZORY SKRÓCONEGO MNOŻENIA

Dla dowolnych liczb rzeczywistych a, b :

$$(a + b)^2 = a^2 + 2ab + b^2$$

$$(a - b)^2 = a^2 - 2ab + b^2$$

$$a^2 - b^2 = (a - b)(a + b)$$

Oraz:

$$\sqrt{2} \cdot \sqrt{3} = \sqrt{6} \qquad 2\sqrt{3} \cdot 4\sqrt{5} = 2 \cdot 4 \sqrt{3 \cdot 5} = 8\sqrt{15}$$

$$\sqrt{5}^2 = \sqrt{5} \cdot \sqrt{5} = 5 \qquad (2\sqrt{5})^2 = 2\sqrt{5} \cdot 2\sqrt{5} = 4 \cdot 5 = 20$$

$$(x^3)^2 = x^{3 \cdot 2} = x^6 \qquad (a^r)^s = a^{r \cdot s} \text{ strona 5}$$

Zad. 1. Zapisz wyrażenie w postaci sumy algebraicznej.

a) $(x + 5)^2 = x^2 + 2 \cdot x \cdot 5 + 5^2 = x^2 + 10x + 25$

b) $(3x - 4)^2 = (3x)^2 + 2(3x)(-4) + (-4)^2 = 9x^2 - 24x + 16$

c) $(2x - \sqrt{3})^2 = (2x)^2 + 2(2x)(-\sqrt{3}) + (-\sqrt{3})^2 = 4x^2 - 4\sqrt{3}x + 3$

d) $(5x^3 + 7)^2 = (5x^3)^2 + 2 \cdot (5x^3) \cdot 7 + 7^2 = 25x^6 + 70x^3 + 49$

e) $(7x - 6)^2 + (6x + 7)^2 = [(7x)^2 + 2(7x)(-6) + (-6)^2] + [(6x)^2 + 2 \cdot 6x \cdot 7 + 7^2] =$
 $= [49x^2 - 84x + 36] + [36x^2 + 84x + 49] =$
 $= \underline{49x^2} - \underline{84x} + \underline{36} + \underline{36x^2} + \underline{84x} + \underline{49} = 85x^2 + 85$

$$\begin{aligned} \text{f) } (9x - 8)^2 - (9x + 8)^2 &= [(9x)^2 + 2 \cdot 9x(-8) + (-8)^2] - [(9x)^2 + 2 \cdot (9x) \cdot 8 + 8^2] = \\ &= [81x^2 - 144x + 64] - [81x^2 + 144x + 64] = \\ &= \underline{81x^2} - \underline{144x} + \underline{64} - \underline{81x^2} - \underline{144x} - \underline{64} = -288x \end{aligned}$$

$$\text{g) } (2x - 7)(2x + 7) = (2x)^2 - 7^2 = 4x^2 - 49$$

$$\text{h) } (3x + 5)(3x - 5) = (3x)^2 - 5^2 = 9x^2 - 25$$

Zad. 2. Oblicz

$$\text{a) } (\sqrt{5} - 7)^2 = (\sqrt{5})^2 + 2 \cdot (\sqrt{5}) \cdot (-7) + (-7)^2 = 5 - 14\sqrt{5} + 49 = 54 - 14\sqrt{5}$$

$$\begin{aligned} \text{b) } (2\sqrt{3} - 4)^2 &= (2\sqrt{3})^2 + 2(2\sqrt{3}) \cdot (-4) + (-4)^2 = 4 \cdot 3 - 16\sqrt{3} + 16 = \underline{12} - 16\sqrt{3} + \underline{16} = \\ &= 28 - 16\sqrt{3} \end{aligned}$$

$$\begin{aligned} \text{c) } (3\sqrt{2} - 2\sqrt{3})^2 &= (3\sqrt{2})^2 + 2(3\sqrt{2})(-2\sqrt{3}) + (-2\sqrt{3})^2 = 9 \cdot 2 - 12\sqrt{6} + 4 \cdot 3 = \\ &= \underline{18} - 12\sqrt{6} + \underline{12} = 30 - 12\sqrt{6} \end{aligned}$$

$$\begin{aligned} \text{d) } (2\sqrt{3} - 4)^2 - (5\sqrt{3} + 2)^2 &= \\ &= [(2\sqrt{3})^2 + 2(2\sqrt{3})(-4) + (-4)^2] - [(5\sqrt{3})^2 + 2(5\sqrt{3}) \cdot 2 + 2^2] = \\ &= [4 \cdot 3 - 16\sqrt{3} + 16] - [25 \cdot 3 + 20\sqrt{3} + 4] = (\underline{12} - 16\sqrt{3} + \underline{16}) - (\underline{75} + 20\sqrt{3} + \underline{4}) = \\ &= (28 - 16\sqrt{3}) - (79 + 20\sqrt{3}) = \underline{28} - \underline{16\sqrt{3}} - \underline{79} - \underline{20\sqrt{3}} = -51 - 36\sqrt{3} \end{aligned}$$

$$\begin{aligned} \text{e) } (3\sqrt{7} - 8)^2 - (2\sqrt{7} + 4)^2 &= \\ &= [(3\sqrt{7})^2 + 2(3\sqrt{7})(-8) + (-8)^2] - [(2\sqrt{7})^2 + 2(2\sqrt{7}) \cdot 4 + 4^2] = \\ &= [9 \cdot 7 - 48\sqrt{7} + 64] - [4 \cdot 7 + 16\sqrt{7} + 16] = \\ &= (63 - 48\sqrt{7} + 64) - (28 + 16\sqrt{7} + 16) = (127 - 48\sqrt{7}) - (44 + 16\sqrt{7}) = \\ &= 127 - 48\sqrt{7} - 44 - 16\sqrt{7} = 83 - 32\sqrt{7} \end{aligned}$$

$$\text{f) } (2\sqrt{3} - 1)(2\sqrt{3} + 1) = (2\sqrt{3})^2 - 1^2 = 4 \cdot 3 - 1 = 12 - 1 = 11.$$

Zad. 3. Usuń niewymierność z mianownika

$$\text{a) } \frac{5}{\sqrt{7}-2} \cdot \frac{\sqrt{7}+2}{\sqrt{7}+2} = \frac{5(\sqrt{7}+2)}{(\sqrt{7})^2-2^2} = \frac{5\sqrt{7}+5 \cdot 2}{7-4} = \frac{5\sqrt{7}+10}{3}$$

$$\begin{aligned} \text{b) } \frac{7}{2\sqrt{3}+4} \cdot \frac{2\sqrt{3}-4}{2\sqrt{3}-4} &= \frac{7(2\sqrt{3}-4)}{(2\sqrt{3})^2-4^2} = \frac{7 \cdot 2\sqrt{3} - 7 \cdot 4}{4 \cdot 3 - 16} = \frac{14\sqrt{3} - 28}{12 - 16} = \\ &= \frac{14\sqrt{3} - 28}{-4} \cdot \frac{(-1)}{(-1)} = \frac{-14\sqrt{3} + 28}{4} = \frac{28 - 14\sqrt{3}}{4} = \frac{14 - 7\sqrt{3}}{2} \end{aligned}$$

$$\begin{aligned} \text{c) } \frac{3+\sqrt{2}}{2\sqrt{2}-4} \cdot \frac{2\sqrt{2}+4}{2\sqrt{2}+4} &= \frac{(3+\sqrt{2})(2\sqrt{2}+4)}{(2\sqrt{2})^2-4^2} = \frac{3 \cdot 2\sqrt{2} + 3 \cdot 4 + \sqrt{2} \cdot 2\sqrt{2} + \sqrt{2} \cdot 4}{4 \cdot 2 - 16} = \\ &= \frac{6\sqrt{2} + 12 + 2 \cdot 2 + 4\sqrt{2}}{8 - 16} = \frac{10\sqrt{2} + 12 + 4}{-8} = -\frac{10\sqrt{2} + 16}{8} = -\frac{5\sqrt{2} + 8}{4} \end{aligned}$$

$$d) \frac{5+2\sqrt{3}}{5+3\sqrt{3}} \cdot \frac{5-3\sqrt{3}}{5-3\sqrt{3}} = \frac{25-15\sqrt{3}+10\sqrt{3}-6 \cdot 3}{5^2 - (3\sqrt{3})^2} = \frac{25-5\sqrt{3}-18}{25-9 \cdot 3} = \frac{7-5\sqrt{3}}{25-27} =$$

$$= \frac{7-5\sqrt{3}}{-2} = \frac{-7+5\sqrt{3}}{2} = \frac{5\sqrt{3}-7}{2}$$

Zad. 4. Wiedząc, że $x + y = 8$ oraz $xy = 5$ oblicz $x^4 + y^4$.

$$(x+y)^2 = x^2 + 2xy + y^2$$

$$8^2 = x^2 + 2 \cdot 5 + y^2$$

$$64 - 10 = x^2 + y^2$$

$$x^2 + y^2 = 54$$

$$(x^2 + y^2)^2 = (x^2)^2 + 2x^2y^2 + (y^2)^2$$

$$(x^2 + y^2)^2 = x^4 + 2(xy)^2 + y^4$$

$$54^2 = x^4 + 2 \cdot 5^2 + y^4$$

$$2916 = x^4 + 2 \cdot 25 + y^4$$

$$2916 - 50 = x^4 + y^4$$

$$x^4 + y^4 = 2866$$

Zad. 5. Usuń niewymierność z mianownika $\frac{2}{4+\sqrt{3}+\sqrt{2}}$

$$\frac{2}{4+\sqrt{3}+\sqrt{2}} \cdot \frac{4+\sqrt{3}-\sqrt{2}}{4+\sqrt{3}-\sqrt{2}} = \frac{2(4+\sqrt{3}-\sqrt{2})}{(4+\sqrt{3})^2 - (\sqrt{2})^2} = \frac{8+2\sqrt{3}-2\sqrt{2}}{4^2 + 2 \cdot 4 \cdot \sqrt{3} + \sqrt{3}^2 - 2} =$$

$$= \frac{8+2\sqrt{3}-2\sqrt{2}}{16+8\sqrt{3}+3-2} = \frac{8+2\sqrt{3}-2\sqrt{2}}{17+8\sqrt{3}} \cdot \frac{17-8\sqrt{3}}{17-8\sqrt{3}} =$$

$$= \frac{136-64\sqrt{3}+34\sqrt{3}-16 \cdot 3-34\sqrt{2}+16\sqrt{6}}{17^2 - (8\sqrt{3})^2} =$$

$$= \frac{136-30\sqrt{3}-48-34\sqrt{2}+16\sqrt{6}}{289-64 \cdot 3} = \frac{88-30\sqrt{3}-34\sqrt{2}+16\sqrt{6}}{289-192} =$$

$$= \frac{88-30\sqrt{3}-34\sqrt{2}+16\sqrt{6}}{97}$$