MateMATURA #1 Rozwiązania

Umiejętność: II.1. Uczeń stosuje wzory skróconego mnożenia na: $(a+b)^2$, $(a-b)^2$, a^2-b^2;

Warto wiedzieć:
Z tablic maturalnych (str. 7)

6. Wzory skróconego mnożenia

Dla dowolnych liczb rzeczywistych a, b:

$$(a+b)^2 = a^2 + 2ab + b^2$$

$$(a-b)^2 = a^2 - 2ab + b^2$$

$$a^2 - b^2 = (a-b)(a+b)$$

oraz:

$\sqrt{2} \cdot \sqrt{3} = \sqrt{6}$ $\quad$ $2\sqrt{3} \cdot 4\sqrt{5} = 2 \cdot 4\sqrt{3 \cdot 5} = 8\sqrt{15}$

$\sqrt{5}^2 = \sqrt{5} \cdot \sqrt{5} = 5$ $\quad$ $(2\sqrt{5})^2 = 2\sqrt{5} \cdot 2\sqrt{5} = 4 \cdot 5 = 20$

$(x^3)^2 = x^{3 \cdot 2} = x^6$ $\quad$ $(a^r)^s = a^{r \cdot s}$ strona 5

Zad. 1. Zapisz wyrażenie w postaci sumy algebraicznej.

a) $(x+5)^2 = x^2 + 2 \cdot x \cdot 5 + 5^2 = x^2 + 10x + 25$

b) $(3x-4)^2 = (3x)^2 + 2(3x)(-4) + (-4)^2 = 9x^2 - 24x + 16$

c) $(2x-\sqrt{3})^2 = (2x)^2 + 2(2x)(-\sqrt{3}) + (-\sqrt{3})^2 = 4x^2 - 4\sqrt{3}x + 3$

d) $(5x^3+7)^2 = (5x^3)^2 + 2 \cdot (5x^3) \cdot 7 + 7^2 = 25x^6 + 70x^3 + 49$

e) $(7x-6)^2 + (6x+7)^2 = [(7x)^2 + 2(7x)(-6) + (-6)^2] + [(6x)^2 + 2 \cdot 6x \cdot 7 + 7^2] =$

$= [49x^2 - 84x + 36] + [36x^2 + 84x + 49] =$

$= \underline{49x^2} - \underline{84x} + 36 + \underline{36x^2} + \underline{84x} + 49 = 85x^2 + 85$

f) $(9x-8)^2-(9x+8)^2 = [(9x)^2+2\cdot 9x(-8)+(-8)^2]-[(9x)^2+2\cdot(9x)\cdot 8+8^2]=$

$=[81x^2-144x+64]-[81x^2+144x+64]=$

$=81x^2-144x+64-81x^2-144x-64=-288x$

g) $(2x-7)(2x+7) = (2x)^2-7^2=4x^2-49$

h) $(3x+5)(3x-5) = (3x)^2-5^2=9x^2-25$

Zad. 2. Oblicz

a) $(\sqrt{5}-7)^2=(\sqrt{5})^2+2\cdot(\sqrt{5})\cdot(-7)+(-7)^2=5-14\sqrt{5}+49=54-14\sqrt{5}$

b) $(2\sqrt{3}-4)^2=(2\sqrt{3})^2+2(2\sqrt{3})\cdot(-4)+(-4)^2=4\cdot 3-16\sqrt{3}+16=12-16\sqrt{3}+16=$

$=28-16\sqrt{3}$

c) $(3\sqrt{2}-2\sqrt{3})^2=(3\sqrt{2})^2+2(3\sqrt{2})(-2\sqrt{3})+(-2\sqrt{3})^2=9\cdot 2-12\sqrt{6}+4\cdot 3=$

$=18-12\sqrt{6}+12=30-12\sqrt{6}$

d) $(2\sqrt{3}-4)^2-(5\sqrt{3}+2)^2=$

$=[(2\sqrt{3})^2+2(2\sqrt{3})(-4)+(-4)^2]-[(5\sqrt{3})^2+2\cdot(5\sqrt{3})\cdot 2+2^2]=$

$=[4\cdot 3-16\sqrt{3}+16]-[25\cdot 3+20\sqrt{3}+4]=(12-16\sqrt{3}+16)-(75+20\sqrt{3}+4)=$

$=(28-16\sqrt{3})-(79+20\sqrt{3})=28-16\sqrt{3}-79-20\sqrt{3}=-51-36\sqrt{3}.$

e) $(3\sqrt{7}-8)^2-(2\sqrt{7}+4)^2=$

$=[(3\sqrt{7})^2+2\cdot(3\sqrt{7})\cdot(-8)+(-8)^2]-[(2\sqrt{7})^2+2\cdot(2\sqrt{7})\cdot 4+4^2]=$

$=[9\cdot 7-48\sqrt{7}+64]-[4\cdot 7+16\sqrt{7}+16]=$

$=(63-48\sqrt{7}+64)-(28+16\sqrt{7}+16)=(127-48\sqrt{7})-(44+16\sqrt{7})=$

$=127-48\sqrt{7}-44-16\sqrt{7}=83-64\sqrt{7}.$

f) $(2\sqrt{3}-1)(2\sqrt{3}+1)=(2\sqrt{3})^2-1^2=4\cdot 3-1=12-1=11.$

Zad. 3. Usuń niewymierność z mianownika

a) $\frac{5}{\sqrt{7}-2}\cdot\frac{\sqrt{7}+2}{\sqrt{7}+2}=\frac{5(\sqrt{7}+2)}{(\sqrt{7})^2-2^2}=\frac{5\sqrt{7}+5\cdot 2}{7-4}=\frac{5\sqrt{7}+10}{3}$

b) $\frac{7}{2\sqrt{3}+4}\cdot\frac{2\sqrt{3}-4}{2\sqrt{3}-4}=\frac{7(2\sqrt{3}-4)}{(2\sqrt{3})^2-4^2}=\frac{7\cdot 2\sqrt{3}-7\cdot 4}{4\cdot 3-16}=\frac{14\sqrt{3}-28}{12-16}=$

$=\frac{14\sqrt{3}-28}{-4}\cdot\frac{(-1)}{(-1)}=\frac{-14\sqrt{3}+28}{4}=\frac{28-14\sqrt{3}}{4}=\frac{14-7\sqrt{3}}{2}$

c) $\frac{3+\sqrt{2}}{2\sqrt{2}-4}\cdot\frac{2\sqrt{2}+4}{2\sqrt{2}+4}=\frac{(3+\sqrt{2})(2\sqrt{2}+4)}{(2\sqrt{2})^2-4^2}=\frac{3\cdot 2\sqrt{2}+3\cdot 4+\sqrt{2}\cdot 2\sqrt{2}+\sqrt{2}\cdot 4}{4\cdot 2-16}=$

$=\frac{6\sqrt{2}+12+2\cdot 2+4\sqrt{2}}{8-16}=\frac{10\sqrt{2}+12+4}{-8}=-\frac{10\sqrt{2}+16}{8}=-\frac{5\sqrt{2}+8}{4}$

d) $\frac{5+2\sqrt{3}}{5+3\sqrt{3}} \cdot \frac{5-3\sqrt{3}}{5-3\sqrt{3}} = \frac{25-15\sqrt{3}+10\sqrt{3}-6\cdot 3}{5^2-(3\sqrt{3})^2} = \frac{25-5\sqrt{3}-18}{25-9\cdot 3} = \frac{7-5\sqrt{3}}{25-27} =$

$= \frac{7-5\sqrt{3}}{-2} = \frac{-7+5\sqrt{3}}{2} = \frac{5\sqrt{3}-7}{2}$

Zad. 4. Wiedząc, że $x + y = 8$ oraz $xy = 5$ oblicz $x^4 + y^4$.

$$(x+y)^2 = x^2+2xy+y^2$$
$$8^2 = x^2+2\cdot 5+y^2$$
$$64-10 = x^2+y^2$$
$$x^2+y^2 = 54$$
$$(x^2+y^2)^2 = (x^2)^2+2x^2y^2+(y^2)^2$$
$$(x^2+y^2)^2 = x^4+2(xy)^2+y^4$$
$$54^2 = x^4+2\cdot 5^2+y^4$$
$$2916 = x^4+2\cdot 25+y^4$$
$$2916-50 = x^4+y^4$$
$$x^4+y^4 = 2866$$

Zad. 5. Usuń niewymierność z mianownika $\frac{2}{4+\sqrt{3}+\sqrt{2}}$

$\frac{2}{4+\sqrt{3}+\sqrt{2}} \cdot \frac{4+\sqrt{3}-\sqrt{2}}{4+\sqrt{3}-\sqrt{2}} = \frac{2(4+\sqrt{3}-\sqrt{2})}{(4+\sqrt{3})^2-(\sqrt{2})^2} = \frac{8+2\sqrt{3}-2\sqrt{2}}{4^2+2\cdot 4\cdot\sqrt{3}+\sqrt{3}^2-2} =$

$= \frac{8+2\sqrt{3}-2\sqrt{2}}{16+8\sqrt{3}+3-2} = \frac{8+2\sqrt{3}-2\sqrt{2}}{17+8\sqrt{3}} \cdot \frac{17-8\sqrt{3}}{17-8\sqrt{3}} =$

$= \frac{136-64\sqrt{3}+34\sqrt{3}-16\cdot 3-34\sqrt{2}+16\sqrt{6}}{17^2-(8\sqrt{3})^2} =$

$= \frac{136-30\sqrt{3}-48-34\sqrt{2}+16\sqrt{6}}{289-64\cdot 3} = \frac{88-30\sqrt{3}-34\sqrt{2}+16\sqrt{6}}{289-192} =$

$= \frac{88-30\sqrt{3}-34\sqrt{2}+16\sqrt{6}}{97}$